

Complete set of electromagnetic corrections to strongly interacting systems

A. N. KVINKHIDZE,* B. BLANKLEIDER

*Department of Physics, The Flinders University of South Australia,
Bedford Park, SA 5042, AUSTRALIA
E-mail: kvinikh@daria.ph.flinders.edu.au*

Abstract

We show how to obtain a complete set of electromagnetic corrections to a given nonperturbative model of strong interactions based on integral equations. The gauge invariance of these corrections is a consequence of their completeness.

I. INTRODUCTION

Electromagnetic corrections to strong interaction models play a crucial role in a number of important problems in nuclear and particle physics. Together with the u - d quark mass difference, electromagnetic corrections are responsible for isospin symmetry breaking and charge symmetry breaking (CSB) in hadronic systems. A manifestation of this for strongly bound hadronic systems is the mass splittings of isomultiplets as for example in the case of the neutron-proton mass difference [1] and the binding energy difference of ${}^3\text{H}$ and ${}^3\text{He}$ [2]. Similarly for hadron-hadron scattering the electromagnetic corrections play an important role in causing isospin symmetry and charge symmetry breaking; indeed there has been an intensive effort at the meson factories to measure CSB in the reactions $\pi N \rightarrow \pi N$, $np \rightarrow np$, $np \rightarrow d\pi^0$, $\pi^\pm d \rightarrow \pi^\pm d$, etc. [3]. Electromagnetic corrections also play an important role in extracting strong interaction quantities from experiments that are sensitive to the presence of the electromagnetic interaction. A current example is the proposed determination of the π - π scattering lengths from measurements of the lifetime of $\pi^+\pi^-$ atoms [4]. Although the instability of this atom is due to the strong interaction process $\pi^+\pi^- \rightarrow \pi^0\pi^0$, the exact lifetime is sensitive to electromagnetic corrections.

The most significant electromagnetic corrections are those of lowest order (e^2) in the electromagnetic coupling constant e . Yet because of the nonperturbative nature of strong interactions, it has not been known how to include electromagnetic interactions in such a way that *all* possible lowest order corrections are included. Without the ability of calculating the complete set of lowest order corrections, any estimates of isospin violation, CSB, or other relevant quantity of interest cannot be considered to be reliable.

*On leave from The Mathematical Institute of Georgian Academy of Sciences, Tbilisi, Georgia.

In this paper we solve this problem by showing how to include the complete set of lowest order electromagnetic corrections into a nonperturbative strong interaction model based on integral equations.

II. ELECTROMAGNETIC CORRECTION TO THE GREEN FUNCTION

We work in the framework of relativistic quantum field theory where the strong interactions of a quark or hadronic system are described by a model potential K . For clarity of presentation we treat all particles as distinguishable; consideration of identical particle symmetry does not affect the method for including electromagnetic interactions presented here and will be discussed elsewhere. Thus the strong interaction Green function G is modelled by solving the integral equation

$$G = G_0 + G_0 K G. \quad (1)$$

Eq. (1) is a symbolic equation where momentum, spin, and isospin variables have been suppressed, and where integrations over intermediate state momenta and sums over internal spins and isospins are also not shown. We shall use such a shorthand notation throughout.

To obtain the complete set of lowest order electromagnetic corrections to the strong interaction Green function G , we need to insert an internal photon with propagator $D_{\mu\nu}$ into G in all possible ways. This can be achieved even though G is given nonperturbatively by a simple extension of the gauging of equations method introduced in Ref. [5]. In this method Eq. (1) is used to write

$$G^\mu = G_0^\mu + G_0^\mu K G + G_0 K^\mu G + G_0 K G^\mu \quad (2)$$

where G^μ is the quantity obtained from G by attaching an external photon everywhere in the strong interaction contributions to G (the quantities G_0^μ and K^μ are similarly obtained from G_0 and K by attaching photons everywhere). Eq. (2) follows from the requirement of coupling an external photon everywhere in Eq. (1). Similarly the requirement of coupling an *internal* photon everywhere in Eq. (1) allows us to write down the equation for the complete set of lowest order electromagnetic corrections to G :

$$\delta G = \delta G_0 + \delta G_0 K G + G_0 \delta K G + G_0 K \delta G + (G_0^\mu K^\nu G + G_0^\mu K G^\nu + G_0 K^\mu G^\nu) D_{\mu\nu} \quad (3)$$

where δG and δG_0 are the complete set of lowest order electromagnetic corrections to G and G_0 respectively. Unlike δG which has internal photons coupled everywhere, δK consists of the strong interaction potential K with all possible photon insertions *except* those that have one or both photon legs attached to an external line. This should be clear from Eq. (3) where such terms, missing from δK , are included in other terms of the equation. Note that the last three terms of Eq. (3) involve photons linking different quantities; for example, the last term has one end of a photon attached to K and the other to G in Eq. (1).

By formally solving Eq. (2) we obtain an explicit expression for G^μ [5]:

$$G^\mu = G \Gamma^\mu G; \quad \Gamma^\mu = \Gamma_0^\mu + K^\mu, \quad (4)$$

where

$$\Gamma_0^\mu = G_0^{-1} G_0^\mu G_0^{-1}. \quad (5)$$

In the case of two strongly interacting particles, $G_0 = d_1 d_2$ where d_i is the dressed propagator for particle i (with dressing due to strong interactions only) and therefore

$$G_0^\mu = d_1^\mu d_2 + d_1 d_2^\mu \quad (6)$$

where d_i^μ is given in terms of the one-particle electromagnetic vertex function Γ_i^μ by

$$d_i^\mu = d_i \Gamma_i^\mu d_i, \quad (7)$$

and

$$\Gamma_0^\mu = \Gamma_1^\mu d_2^{-1} + d_1^{-1} \Gamma_2^\mu \quad (8)$$

is the electromagnetic current of the two-particle system in impulse approximation. K^μ in Eq. (4) corresponds to the interaction current and can also be obtained by the gauging of equations method if K is given nonperturbatively by an integral equation. In any case, here we shall treat K^μ as an input quantity that has previously been constructed.

Eq. (3) is an integral equation for the electromagnetic corrections δG which can be formally solved in just the same way that Eq. (2) was solved to get Eq. (4). We obtain that

$$\delta G = G \Delta G \quad (9)$$

where

$$\Delta = \delta K + G_0^{-1} \delta G_0 G_0^{-1} + (\Gamma^\mu G \Gamma^\nu - \Gamma_0^\mu G_0 \Gamma_0^\nu) D_{\mu\nu}. \quad (10)$$

This is the central result of our paper. It expresses the complete set of lowest order electromagnetic corrections to G in terms of the solution to the initial strong interaction problem (G itself). As discussed below, Δ forms the complete set of electromagnetic corrections to the potential K . Because of this completeness, calculations of physical quantities using Eq. (10) will be gauge invariant.

The various contributions to Δ are illustrated diagrammatically in Fig. 1 for the two-body case (note however that Eq. (10) is valid for any number of quarks or hadrons). To reveal the contributions to $G_0^{-1} \delta G_0 G_0^{-1}$ we write

$$\delta G_0 = \delta(d_1 d_2) = \delta d_1 d_2 + d_1 \delta d_2 + G_0 K^\gamma G_0 \quad (11)$$

where δd_i is the complete set of electromagnetic corrections to d_i and K^γ is the one photon exchange potential as illustrated in the first diagram of Fig. 1(b). As the dressed quark or hadron propagator d is expressed in terms of the integral equation

$$d = d_0 + d_0 \Sigma d \quad (12)$$

where d_0 is the bare propagator and Σ is the (strong interaction) dressing term, we can determine δd in just the same way that δG was determined from Eq. (1). We obtain that

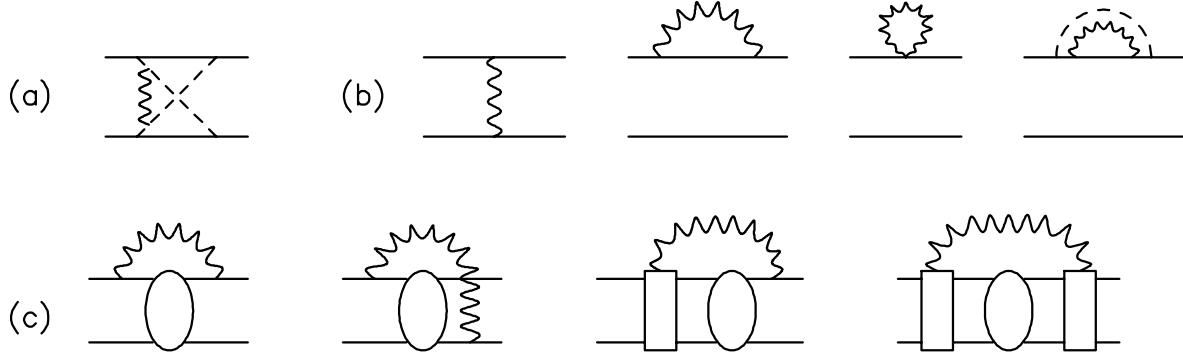


FIG. 1. Diagrams contributing to the electromagnetic correction Δ of Eq. (10). (a) An example of a correction belonging to δK for the case where solid lines are nucleons and dashed lines are pions. (b) The diagrams contributing to $G_0^{-1}\delta G_0 G_0^{-1}$: one-photon exchange, two types of particle dressing by photon, and an example of a correction belonging to $\delta \Sigma$. (c) Contributions to $(\Gamma^\mu G \Gamma^\nu - \Gamma_0^\mu G_0 \Gamma_0^\nu) D_{\mu\nu}$: the oval represents the strong interaction Green function G (with G_0 subtracted in the first two diagrams), while the rectangle with attached photon represents K^μ . All photon-particle vertices are dressed (by strong interactions).

$$d^{-1}\delta d d^{-1} \equiv \Sigma^\gamma = \delta \Gamma_0 + \delta \Sigma + \Gamma^\mu d \Gamma^\nu D_{\mu\nu} \quad (13)$$

where $\delta \Gamma_0$ is defined by the diagram in Fig. 2(a) and corresponds to the term $\Gamma_0^{\mu\nu} D_{\mu\nu}$ where $\Gamma_0^{\mu\nu}$ arises from the gauging of Γ_0^μ . Note that $\delta \Gamma_0 = 0$ for spin 1/2 particles but is not zero, for example, for spin 0 particles. Diagrams contributing to the last two terms of Eq. (13) are also illustrated in Fig. 2. Thus

$$G_0^{-1}\delta G_0 G_0^{-1} = K^\gamma + \Sigma_1^\gamma d_2^{-1} + \Sigma_2^\gamma d_1^{-1} \quad (14)$$

as illustrated in Fig. 1(b).

The last term of Eq. (10), $\Gamma_0^\mu G_0 \Gamma_0^\nu D_{\mu\nu}$, is made up of the first two diagrams of Fig. 1(b). Its role in Eq. (10) is clearly to subtract those contributions from $\Gamma^\mu G \Gamma^\nu D_{\mu\nu}$ which already appear in $G_0^{-1}\delta G_0 G_0^{-1}$. It follows that $(\Gamma^\mu G \Gamma^\nu - \Gamma_0^\mu G_0 \Gamma_0^\nu) D_{\mu\nu}$ is just the connected part of $\Gamma^\mu G \Gamma^\nu D_{\mu\nu}$ minus the one-photon exchange contributions - as illustrated in Fig. 1(c).

As noted previously, δK does not have photons attached to external legs. On the other hand, if we define the corrected Green function as

$$G' \equiv G + \delta G = G + G \Delta G, \quad (15)$$

we see that to order e^2 in the electromagnetic interaction

$$G' \approx (G^{-1} - \Delta)^{-1} = (G_0^{-1} - K - \Delta)^{-1} \quad (16)$$

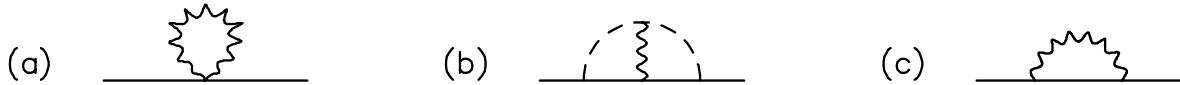


FIG. 2. Diagrams contributing to the complete set of electromagnetic corrections of a single quark or hadron - see Eq. (13): (a) $\delta \Gamma_0$, (b) an example of a contribution to $\delta \Sigma$ in the case where the solid line is a nucleon and the dashed line is a pion, and (c) $\Gamma^\mu d \Gamma^\nu D_{\mu\nu}$.

indicating that Δ can be considered as the full electromagnetic correction to the potential K . Defining $K' = K + \Delta_c$ where Δ_c is the connected part of Δ , it is easy to see that

$$G' = G'_0 + G'_0 K' G' \quad (17)$$

where $G'_0 = d'_1 d'_2$ with $d' = d + d\Sigma^\gamma d'$ being the fully dressed one-particle propagator with all second order electromagnetic interactions included, and $K' = K + \delta K + K^\gamma + (\Gamma^\mu G \Gamma^\nu - \Gamma_0^\mu G_0 \Gamma_0^\nu) D_{\mu\nu}$ is the corrected potential incorporating the complete set of (connected) second order electromagnetic corrections, including those where photons are attached to external legs. The gauge invariance of on-mass-shell amplitudes corresponding to G' is therefore guaranteed (an explicit proof will be given later in a more detailed article).

III. ELECTROMAGNETIC CORRECTIONS TO THE BOUND STATE MASS

The strong interaction bound state wave function ψ satisfies the bound state equation

$$\psi_P = G_0 K \psi_P \quad (18)$$

where the total momentum P is on mass shell: $P^2 = M^2$. The bound state wave function also satisfies the normalization condition

$$i\bar{\psi}_P \frac{\partial}{\partial P^2} (G_0^{-1} - K) \psi_P = 1 \quad (19)$$

where the derivative is evaluated at $P^2 = M^2$. The mass M of this bound state involves strong interactions only and will change by some amount δM on inclusion of the electromagnetic interaction. Our goal is to find δM for the case where all possible electromagnetic interactions are included up to order e^2 .

With the full Green function including electromagnetic interactions given by Eq. (17), the bound state equation becomes

$$\psi'_{P'} = G'_0 K' \psi'_{P'} \quad \Rightarrow \quad (G_0^{-1} - K - \Delta) \Big|_{P'^2=M'^2} \psi'_{P'} = 0 \quad (20)$$

where $\psi'_{P'} = \psi_P + \delta\psi_P$ is the wave function with electromagnetic corrections included, and $P'^2 = M'^2$ where $M' = M + \delta M$ is the corresponding bound state mass. Making an expansion up to order e^2 around the point $P^2 = M^2$ and using both Eq. (18) and Eq. (19) we obtain that

$$\delta M = \frac{1}{2M} \bar{\psi}_P \Delta \psi_P \quad (21)$$

with the right hand side evaluated at $P^2 = M^2$. The gauge invariance of the mass correction δM as given by Eq. (21) follows from the completeness of the included electromagnetic interactions, although again we shall leave an explicit proof to a more detailed publication.

The authors would like to thank H. Leutwyler and M.P. Locher for their very informative comments and the Australian Research Council for their financial support.

REFERENCES

- [1] G.J. Stephenson, Jr., K. Maltman, and T. Goldman *Phys. Rev. D* **43**, 860 (1991).
- [2] Y. Wu, S. Ishikawa, and T. Sasakawa, *Phys. Rev. Lett.* **64**, 1875 (1990); *erratum* **66**, 242 (1991).
- [3] G.A. Miller and W.H.T. van Oers, in *Symmetries and fundamental interactions in nuclei*, p. 127, Ed. W.C. Haxton and E.M. Henley (World Scientific, Singapore, 1995).
- [4] M.A. Ivanov, V.E. Lyubovitskij, E.Z. Lipartia, and A.G. Rusetsky, hep-ph/9805356; see also <http://www.cern.ch/DIRAC/>.
- [5] A.N. Kvinikhidze and B. Blankleider, *Nucl. Phys.* **A631**, 559c (1998); see also B. Blankleider and A.N. Kvinikhidze, *Unified model of the relativistic πNN and $\gamma\pi NN$ systems*, APCTP Workshop on Astro-Hadron Physics , Seoul, Korea, 25-31 October, 1997, to be published by World Scientific, nucl-th/9802017.